ABSTRACTS OF LECTURES
ARITHMETIC AND ALGEBRAIC GEOMETRY 2015

Venue: Graduate School of Mathematical Sciences, The University of Tokyo
Dates: January 27 (Tues) - 31 (Sat), 2015

N. Aoki (Rikkyo Univ., Japan)
Algebraic cycles on Fermat varieties

Abstract. In this talk we will survey algebraic cycles on Fermat varieties focusing on the work of Shioda and myself. We will also discuss some new results on the generators of the Neron-Severi group of the Fermat surface of degree 12.

M. Asakura (Hokkaido Univ., Japan)
Period conjecture of Gross-Deligne for fibrations

Abstract. The period conjecture of Gross-Deligne asserts that the periods of a CM motive are written in a certain product of Gamma values which is determined by the Hodge structure. We show it for cohomology of fibrations with multiplication arising from relative algebraic correspondences.

L. Illusie(Paris XI, France)
Around the Thom-Sebastiani theorem

Abstract. For germs of holomorphic functions $f : \mathbf{C}^{m+1} \to \mathbf{C}$, $g : \mathbf{C}^{n+1} \to \mathbf{C}$ having an isolated critical point at 0 with value 0, the classical Thom-Sebastiani theorem describes the vanishing cycles group $\Phi^{m+n+1}(f \oplus g)$ (and its monodromy) as a tensor product $\Phi^m(f) \otimes \Phi^n(g)$, where $(f \oplus g)(x, y) = f(x) + g(y), x = (x_0, ..., x_m), y = (y_0, ..., y_n)$. I will discuss algebraic variants and generalizations of this result over fields of any characteristic, where the tensor product is replaced by a certain local convolution product, as suggested by Deligne. The main theorem is a Künneth formula for $R\Psi$ in the framework of Deligne's theory of nearby cycles over general bases.

G. van der Geer(Amsterdam Univ., Netherlands)
Vector-valued modular forms of low genus.

Abstract. Vector-valued modular forms are the natural generalization of elliptic modular forms. We explain the relation with the cohomology of local systems on moduli spaces of curves and sketch some recent results. This is joint work with Bergstroem and Faber and with Clery.

A. Ichino (Kyoto Univ., Japan)
The Gross-Prasad conjecture and local theta correspondence.

Abstract. We discuss two restriction problems in the representation theory of reductive groups over p-adic fields. One is the local Gross-Prasad conjecture concerning the restriction of representations of a unitary group to a smaller unitary group, and the other is local theta correspondence concerning the lifting of representations of a unitary group to another unitary group using branching laws. We prove the Fourier-Jacobi case of the local Gross-Prasad conjecture as well as two conjectures of D. Prasad concerning local theta correspondence, which explicitly describe the restriction problems. This is joint work with Wee Teck Gan.

T. Katsura (Hosei Univ., Japan)
On a 1-dimensional family of Enriques surfaces in characteristic 2.

Abstract. We construct a 1-dimensional family of classical and supersingular Enriques surfaces in characteristic 2 covered by the supersingular $K3$ surface with Artin invariant 1. We also show that there exist 30 nonsingular rational curves on each Enriques surface which make a beautiful configuration. This is a joint work with S. Kondo.

J. H. Keum (KIAS, Korea)
Moduli space of Enriques surfaces with a fixed ADE-configuration.

Abstract. Enriques surfaces form a 10-dimensional space. Nodal Enriques surfaces, i.e., Enriques surfaces with A_1-configuration, form a 9-dimensional subspace. In general, the moduli space of Enriques surfaces with a fixed ADE-configuration R has codimension at least $\mathrm{rank}(R)$. In this talk I will focus on the case of $\mathrm{rank}(R) = 9$, the maximum possible.

S. Kimura (Hiroshima Univ., Japan)
Rationality and Irrationality of (infinitesimal) motivic Chow Series.

Abstract. Motivic zeta of algebraic varieties modulo rational equivalence is conjectured to be rational, and is related to Bloch-Beilinson-Murre conjecture. The rationality of Motivic Chow Series, a higher dimensional analogy, is more mysterious and subtle, and the rationality depends on the varieties and the equivalence relations. In this talk, we discuss some rationality and irrationality results, including infinitesimal equivalence relation cases.

A. Kumar (M.I.T, USA)
Moduli spaces of abelian surfaces via elliptic K3 surfaces with Shioda-Inose structure.

Abstract. Shioda-Inose structures were introduced in connection with Piatetski-Shapiro and Shafarevich's proof of the Torelli theorem for algebraic K3 surfaces over the complex numbers; they were key to parametrizing the (discrete) moduli space of singular K3 surfaces. The same device has more recently been used to explicitly describe moduli spaces of abelian surfaces with extra endomorphisms, via elliptic K3 surfaces of high Picard number (but less than 20). I will describe these developments, starting with an identification of A_2 with the moduli space of elliptic K3 surfaces with singular fibers of type E_8 and E_7.

M. Kurihara (Keio Univ., Japan)
Arithmetic of zeta elements for the Tate motive.

Abstract. This is a joint work with David Burns and Takamichi Sano. For an equivariant zeta function for the Tate motive (Artin L-functions) over a number field, we introduce zeta elements which are algebraic incarnations of zeta values, and discuss their arithmetic properties, especially the Galois module structure of certain Weil etale cohomology groups and of ideal class groups, and relations with several conjectures.

M. Kuwata (Chuo Univ., Japan)
Elliptic K3 surfaces with Mordell-Weil rank 18.

Abstract. The rank of the Mordell-Weil group of an elliptic K3 surface over a field of characteristic 0 can be as high as 18. Examples of such elliptic K3 surfaces can be constructed from isogenous pairs of elliptic curves with complex multiplication. More precisely, they are constructed as finite coverings of the Kummer surface associated with the product of such pairs of elliptic curves. The determination of the rank of

the Mordell-Weil group for such elliptic surfaces depends on a theorem of Inose, which relies on a transcendental argument, and we have not had a good understanding of the structure of the Mordell-Weil group itself. In this talk we show the field of definition of the Mordell-Weil groups of such elliptic surfaces and give an explicit description of their generators. This is joint work with Abhinav Kumar.

Ch. Liedtke (Tech. Univ. München, Germany)
Supersingular K3 Surfaces are Unirational.

Abstract. We show that supersingular K3 surfaces are related by purely inseparable isogenies. As an application, we deduce that they are unirational, which confirms conjectures of Artin, Rudakov, Shafarevich, and Shioda. The main ingredient in the proof is to use the formal Brauer group of a Jacobian elliptically fibered K3 surface to construct a family of "moving torsors" under this fibration that eventually relates supersingular K3 surfaces of different Artin invariants by purely inseparable isogenies.

K. Oguiso (Osaka Univ., Japan)
Automorphisms of elliptic K3 surfaces and Salem numbers of maximal degree.

Abstract. We explain one way to produce automorphisms of K3 surface of positive entropy from Mordell-Weil groups. Combining this with Professor Shioda's results on elliptic fibration with maximal rank, we show that any supersingular K3 surface of Artin invariant 1 in characteristic $p \neq 5, 7, 13$ has an automorphism the entropy of which is the natural logarithm of a Salem number of the maximal degree 22, in particular non-liftable ones to characteristic zero. This is a joint work with Professor Hélène Esnault and Doctor Xun Yu.

S. Saito (Tokyo Inst. Tech., Japan)
K-theory of rigid spaces and topological Chow groups of algebraic varieties.

Abstract. Let K be a complete discretely valued field and R be the ring of integers with a prime element π. Let X be a (formal) scheme over R and write $X_n = X \otimes_R R/(\pi^{n+1})$ for $n \geq 0$. The *continuous* K-groups of X are defined as

$$K_i^{cont}(X) := \operatorname{proj}\lim n K_i(X_n) \quad (i \in \mathbf{Z}),$$

where $K_i(X_n)$ are the higher algebraic K-groups of X_n defined by Quillen and others. Recently Bloch-Esnault-Kerz and Beilinson studied this and succeeded in proving the deformational part of Grothendieck's variational Hodge conjecture and its p-adic version due to Fontaine-Messing.

Motivated by this, we introduce new topological K-theory for rigid spaces over K which are closely related to the continuous K-theory of their formal models over R. We also introduce "topological higher Chow groups" for rigid spaces which are conjecturally related to the topological K-groups in the same way as the case of algebraic varieties. The topological Chow groups of a rigid space associated to an algebraic variety over K are described as the groups of cycles modulo new equivalence which refines algebraic equivalence. This is a joint work in progress with Moritz Kerz and Georg Tamme.

T. Saito (Univ. Tokyo, Japan)
The characteristic cycle and the singular support of an etale sheaf.

Abstract. We define the characteristic cycle of an etale sheaf on a smooth variety of arbitrary dimension in positive characteristic assuming the existence of singular support satisfying certain local acyclicity conditions. It satisfies a Milnor formula for vanishing cycles and an index formula for the Euler-Poincare characteristic. This is a generalization to higher dimension of the results for surfaces discussed last year.

M. Schuett (Leibniz Univ. Hannover, Germany)
Profile of some beautiful K3 surfaces.

Abstract. Inspired by work of Shioda, and drawing on joint efforts, I will discuss the geometry and arithmetic of some particular K3 surface. I will emphasise a number of topical questions such as elliptic fibrations, Mordell-Weil groups including explicit generators, automorphisms, and time permitting, dynamics.

I. Shimada (Hiroshima Univ., Japan)
Holes of the Leech lattice and projective models of K3 surfaces

Abstract. By means of the structure of the Voronoi cell of the Leech lattice, we give an effective bound for projective models of a K3 surface of a fixed degree modulo the automorphism group of the K3 surface.

T. Shioda (Rikkyo Univ., Japan)
Mordell-Weil lattice of higher genus fibration on a Fermat surface

Abstract.

We study the Mordell-Weil lattice (MWL) of a higher genus fibration, called the axial fibration, on the Fermat surface X_m of degree m:

$$x_0^m + x_1^m + x_2^m + x_3^m = 0. \tag{1}$$

For example, if m is an odd integer > 3 and if l_0 is the line on X_m

$$l_0 = \{x_0 + x_1 = 0, \quad x_2 + x_3 = 0\}, \tag{2}$$

then the axial fibration on X_m with axis l_0

$$f : X_m \to \mathbf{P}^1 \tag{3}$$

is defined by the map

$$f(x) = t = -\frac{x_2 + x_3}{x_0 + x_1} = -\frac{\Pi'(x_0 + \zeta^{n-1} x_1)}{\Pi'(x_2 + \zeta^{n-1} x_3)} \tag{4}$$

where $\zeta = e^{\frac{2\pi i}{m}}$ and Π' means the product from $n = 2$ to m.

We obtain the basic theorems (the Mordell-Weil rank, the height formula, etc), and give some examples. For example, if $m = 5$, then we have genus 3 fibration, and we show that the Mordell-Weil group M has rank $r = 19$ and it is generated by the rational points (sections) corresponding to the lines on X_5. Moreover we prove that M has the torsion subgroup of order two, by studying the height matrix of the MWL in question.

T. Terasoma (Univ. Tokyo, Japan)
Mixed elliptic motives and depth filtration of multiple zeta values

Abstract. For a sequence of integers $n = (n_1, \ldots, n_r)$, we define a multiple zeta value $\zeta(n_1, \ldots, n_r)$. The number r is called the depth of the index n. The depths define a filtration on the Q-subspace of R generated by multiple zeta values: Broadhurst and Kreimer conjectured an explicit formula for the double generating function arising from weights and depths. In this talk, we give an interpretation of this conjecture in terms of mixed elliptic motives.

J. Top (Groningen, Netherlands)
First order differential equations.

Abstract. In 1980 Michihiko Matsuda published a book presenting an algebraic approach to classical first order differential equations as studied by numerous people in the 19th and early 20th century. Recently we extended some of this theory, in particular addressing algorithmic issues, and the case of positive characteristic. Among other things this led to an analog for first order algebraic equations of the Grothendieck-Katz p-curvature conjecture. This is joint work with Marius van der Put.

E. Urban (Columbia Univ., USA)
p-adic families of automorphic periods and p-adic L-functions.

Abstract. I will explain certain construction of p-adic distributions of automorphic periodson certain unitary groups and some conjectures on these. I will speak about progresses towards those and their consequences for the study of higher rank Selmer groups.