Leibniz Universität Hannover
Fakultät für Mathematik und Physik
Institut für Analysis

SEMINARANKÜNDIGUNG

für das Wintersemester 2013

Thema: Determinants

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Voraussetzungen: Grundlagen zu linearen Operatoren in Hilberträumen, Funktionentheorie, Mannigfaltigkeiten und ggf. Liegruppen.

Überblick:

From the connection between the trace of a matrix with scalar coefficients and its eigenvalues, one can derive a relation between the trace and the determinant of a matrix, namely

$$\det(A) = \exp(\operatorname{tr}(\log A)). \tag{1}$$

At the level of Lie groups, a trace on a Lie algebra is the derivative of the determinant at the identity on the associated Lie group, and with an exponential mapping one recovers in this setting the relation (1).

On trace–class operators over a separable Hilbert space one can promote the trace on matrices to an operator trace. Further generalizing to classical pseudodifferential operators one can consider traces on such operators, and then it is possible to describe determinants on the associated Lie group. In particular, the resulting determinants are multiplicative, a property which does not hold anymore for determinants that are defined by a regularization procedure, giving rise to a multiplicative anomaly.

In mathematics and theoretical physics, regularization is a method that assigns finite values to divergent sums or products, and in particular can be used to define determinants and traces of some operators.

The Fredholm determinant is a complex–valued function, defined for bounded operators on a Hilbert space which differ from the identity operator by a trace–class operator. There is a way of regularizing this determinant so that it can be extended to operators which differ from the identity operator by an element in a Schatten ideal.

Another way of regularizing the ordinary determinant is based on the fact that, for matrices, the determinant is equal to the product of its eigenvalues. For a differential operator A such a product diverges because its eigenvalues increase without bound. In this case, we can consider the zeta function of A, which is a meromorphic function and its differential at the origin gives the determinant of the operator

$$\det_\zeta(A) = e^{-\zeta_A'(0)}. \tag{2}$$

This determinant was introduced by Ray and Singer on the mathematical side and first used by Hawkings on the physics side in order to make sense of partition functions.

The purpose of this seminar is to understand different ways to define the functional determinant of an operator, and its applications in Index Theory, and in Quantum Field Theory when regularizing path integrals.

Literatur:

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