Generalized Monodromy Conjecture in dimension two

We report on the following joint work with A. Némethi. We extend the Monodromy Conjecture of Denef and Loeser in dimension two, incorporating zeta functions with differential forms and targeting *all* monodromy eigenvalues, and also considering singular ambient spaces. That is, we treat in a conceptual unity the poles of the (generalized) topological zeta function and the monodromy eigenvalues associated with an analytic germ $f : (X, 0) \to (\mathbb{C}, 0)$ defined on a normal surface singularity $(X, 0)$. We target the 'right' extension in the case when the link of $(X, 0)$ is a homology sphere. As a first step, we prove a splice decomposition formula for the topological zeta function $Z(f, \omega; s)$ for any f and analytic differential form ω, which will play the key technical localization tool in the later definitions and proofs.

Then, we define a set of 'allowed' differential forms via a local restriction along each splice component. For plane curves we show the following three guiding properties: (1) if s_0 is any pole of $Z(f, \omega; s)$ with ω allowed, then $\exp(2\pi i s_0)$ is a monodromy eigenvalue of f, (2) the 'standard' form is allowed, (3) every monodromy eigenvalue of f is obtained as in (1) for some allowed ω and some s_0. For general $(X, 0)$ we prove (1) unconditionally, and (2)–(3) under an additional (necessary) assumption, which generalizes the semigroup condition of Neumann–Wahl.